

# Slow noise processes in superconducting resonators

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Slow noise processes, with characteristic timescales  $\sim 1$ s, have been studied in planar superconducting resonators. A frequency locked loop is employed to track deviations of the resonator centre frequency with high precision and bandwidth. Comparative measurements are made in varying microwave drive, temperature and between bare resonators and those with an additional dielectric layer. All resonators are found to exhibit flicker frequency noise which increases with decreasing microwave drive. We also show that an increase in temperature results in a saturation of flicker noise in resonators with an additional dielectric layer, while bare resonators stop exhibiting flicker noise instead showing a random frequency walk process.

Slow fluctuations in charge sensitive devices have been frequently examined over the past few decades[1]. Recently, their effects were indirectly observed in superconducting qubits[2] with supporting theoretical work[3] linking them to the presence of two level fluctuators (TLFs)[4]. We present measurements directly probing these slow noise processes in superconducting resonators using a high bandwidth feedback technique with Hz level resolution[5]. Feedback maintains a lock to the resonator centre frequency indefinitely, providing a direct measure of the nature of slow fluctuations and their behavior in varying temperature, microwave drive and TLF density.

Theoretical work[3] suggests that "slow" fluctuators can have a profound -but indirect- effect on the noise and losses in devices operating at microwave frequencies such as qubits. In simplified terms the model considers two distinct ensembles of TLFs: a coherent population that couple directly to the device, and a "slow" population which perturbs the coherent TLFs, by changing their tunnel splitting. Hence, whereas the coherent processes is expected to have short time constants, they are in turn effectively being modulated by processes that can have time constants of the order of seconds or even hours.

Motivated by QIP applications, recent studies on superconducting resonators have focused heavily on sources of dissipation. Measurements have e.g. evaluated the effects of magnetic fields[6] and vortex motion[7]. The remaining dissipation is usually attributed to the presence of two level fluctuators (TLFs). The exact nature of TLFs remains contentious, but a variety of experiments have studied their effects[8–12] and recent models evaluated the effect of TLFs in varying locations[13].

In this paper we study slow noise processes in low loss Niobium (Nb) on sapphire resonators[11]. Resonators are by their very nature sensitive probes: a sudden change in the environment will produce a change in the centre frequency  $\nu_0$  of the resonator; making them ideal devices for studying noise. However, measuring this noise can be

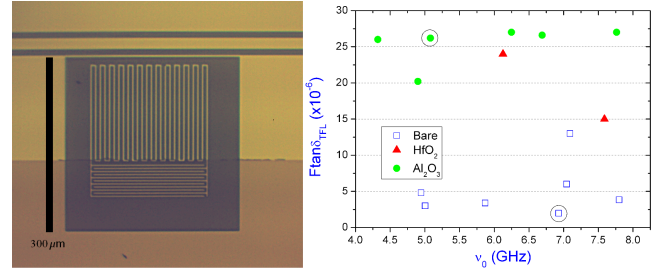


Figure 1. Left: Photograph of device, consisting of a long meandering inductive part and an inter-digitated capacitor. For covered resonators the capacitive part is coated by the additional dielectric layer (red strip). Right: Scatter plot comparing the loss tangents of bare resonators with those covered by  $\text{Al}_2\text{O}_3$  and  $\text{HfO}_2$ . Plot highlights density of TLF's to be uniform with frequency and for covered resonators to have a loss tangent between 5-10 times higher than the bare resonator. Circled is a bare resonator, B1, at 6.93 GHz and an  $\text{Al}_2\text{O}_3$  covered resonator, A1, at 5.08 GHz which are featured in later measurements.

difficult due to the extrinsic low frequency noise present in equipment such as amplifiers and mixers[14]. Here we overcome this problem by using a high-bandwidth measurement method based on a so-called Pound loop which operates at an offset frequency well above the extrinsic flicker corner frequency[5]. Additionally, by depositing an further dielectric layer on top of the resonator we study the effects of an increased TLF density.

Our samples consist of several lumped element resonators coupled to a common feed line (see fig. 1). Transmission through such resonators is described by  $S_{21} = 2[2 + \frac{g}{1+2jQ_l x}]^{-1}$  where  $Q_l$  is the loaded quality factor (defined as the center frequency,  $\nu_0$ , divided by the bandwidth,  $\Delta\nu$ ,  $Q_l = \nu_0/\Delta\nu$ ),  $x$  is the fractional frequency shift  $x = (\nu - \nu_0)/\nu_0$  and  $g$  is the coupling parameter. The center frequency is defined by  $\nu_0 = (2\pi\sqrt{(L + L_K)C})^{-1}$  where  $C$  is the capacitance,  $L$  the inductance and  $L_K$  the kinetic inductance.  $L_K$  varies with the penetration depth and hence with temperature and magnetic field[6]

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Experimentally it is found that as temperature is reduced a monotonic increase in center frequency is observed, while the quality factor increases due to reducing conductor losses as described by Mattis-Bardeen theory[15]. At temperatures much below the superconducting critical temperature,  $T_c$ , these mechanisms saturate. When lowering the temperature beyond this saturation a further change in both Q and center frequency is observed, which is well described in the theory of TLFs [16] [17], by modeling a single TLF as a dipole that can shift states in an asymmetric well by thermally activated tunneling or absorption of resonant photons. The former effect leads to a change in the center frequency while the latter manifests as a power dependent Q. From TLF theory the change in permittivity can be described by

$$\frac{\Delta\epsilon(T)}{\epsilon(T_0)} = -\frac{2nd^2}{3\epsilon} \left( \ln\left(\frac{T}{T_0}\right) - [g(T, \omega) - g(T_0, \omega)] \right) \quad (1)$$

Where  $\epsilon$  is the permittivity,  $n$  the density of TLF states,  $d$  the dipole moment,  $T$  the temperature,  $T_0$  is a reference temperature and  $g(T, \omega) = \text{Re}\Psi(\frac{1}{2} + \hbar\omega/2\pi ik_B T)$  and  $\Psi$  is the complex digamma function. Changes in permittivity relate to frequency changes by  $\frac{\Delta\nu_0}{\nu_0} = -\frac{F}{2} \frac{\Delta\epsilon}{\epsilon}$ . With F being a geometric filling factor.

Equation (1) can be used to determine the *intrinsic* loss tangent  $F\delta_{TLF}^0 = F\frac{2nd^2}{3\epsilon}$  of a resonator, which is proportional to the density of TLFs,  $n$ , and their dipole moment  $d^2$ . This can differ slightly from the loss tangent determined by power dependent Q measurements, due to the former including the effect of non-resonant TLFs [18]. The presence of two distinct populations of TLF -"slow" and coherent- would further remove the direct correlation between loss and noise in the resonator; the indirect effect of slow fluctuators means that they can influence the noise level of the resonator without affecting parameters such as the quality factor.

A He<sup>3</sup> – He<sup>4</sup> dilution refrigerator is used to measure samples, the input signal is attenuated by 50dB and filtered by a 3.5 GHz high pass filter. The output signal is amplified by a cryogenic InP HEMPT amplifier with a noise temperature of 5K and gain of 30dB before further amplification at room temperature. The Resonators consist of 200nm thick niobium films deposited by RF sputtering on R-plane sapphire. To study the effects of TLFs some resonators are left bare, while others have a 50nm layer of Al<sub>2</sub>O<sub>3</sub> deposited by atomic layer deposition (ALD) over the inter-digitated capacitors (see Fig 1). The intrinsic loss tangent is measured by varying the temperature of the copper cold finger between 50 mK and 800 mK. The frequency shift is tracked by a Pound loop (see ref[5]) and fit to equation (1) to extract the intrinsic loss tangent. Figure 1 shows the additional dielectric coating to consistently produce an increased loss tangent by a factor of 5-10 independent of resonance frequency. Circled is bare resonator, B1, and covered resonator, A1, which are studied further below.

The Pound loop uses phase modulation at 1 MHz

which is above the 1/f corner frequency of the amplifier/mixer chain. Feedback is used to establish a frequency locked loop which tracks the center frequency of the resonator in real time within the loop bandwidth ( $\sim 6$  kHz). Readout of the feedback signal allows the centre frequency to be resolved down to the  $\ll 10$  Hz level, ie. fractional frequency resolution of 2 parts in  $10^9$  for a 5 GHz resonator. Discriminating such small frequency shifts provides a vast improvement to both speed and accuracy of loss tangent measurements (using eq. (1)). In directly tracking the centre frequency shift without need of fitting resonance data we make a significant reduction in measurement time. However, more interestingly, the high bandwidth of the feedback loop means the intrinsic frequency jitter of the resonator can also be directly measured by monitoring  $\nu_0$  vs. time.

Processing of the feedback can be performed in either the time or frequency domains. Frequency domain analysis produces root spectral density plots ( $\sqrt{S_y}$  in units of Hz/ $\sqrt{\text{Hz}}$ ) however spectral analysis is not ideal for evaluating low frequency noise due to the numerical pole at 0 and windowing effects[14] (See Supplemental Material at [URL] for [spectral analysis]). Instead Time domain analysis is performed by the simple verifiable technique of calculating the Allan deviation ( $\sigma_y(\tau) = \sqrt{\frac{1}{2} \langle (y_{n+1} - y_n)^2 \rangle}$  in units of Hz) which is similar to the standard deviation except that it converges for all noise processes. It hence serves as a measurement of the frequency jitter for a given measurement time. Both techniques describe noise processes obeying a power law, where for example flicker frequency noise is described by  $\sqrt{S_y} \propto 1/f^{0.5}$  and  $\sigma_y(\tau) \propto \tau^0$ , see ref[5] for more information. A strong advantage of Allan analysis is to determine the time scales over which system noise, e.g. from wide band amplifiers dominates the noise, hence the Allan analysis determines the actual resolution obtainable within a given measurement bandwidth.

A dielectric resonator of diameter 14 mm and height 8 mm supported by a 14 mm post, with  $Q_l \approx 10^5$  and  $\nu_0=8.1$  GHz is first measured as a reference to determine the noise contributed from the electronics in the loop itself. Such resonators are known to be very stable and more importantly do not exhibit flicker frequency noise[19], instabilities instead arise due to thermal or mechanical fluctuations and appear as a linear frequency drift. Superconducting resonators are always at least as noisy as the dielectric resonator, in figure 2 the system noise floor is labeled as the dielectric reference resonator and shown by the pink hashed region. The frequency jitter of a bare (covered) resonator, B1 (A1), with  $Q_l \approx 35000$ ,  $\nu_0=6.93$  GHz and  $F\delta_{TLF}^0=2.0 \times 10^{-6}$  ( $Q_l \approx 30000$ ,  $\nu_0=5.08$  GHz and  $F\delta_{TLF}^0=2.6 \times 10^{-5}$ ) is shown in figure 2a (figure 2c).

The power within a resonator can be estimated by  $P_{circ} = P_{inc} Q_L 10^{-IL/20}/\pi$ , where  $P_{inc}$  is the power in watts incident on the resonator,  $Q_L$  the loaded Q and IL the insertion loss in dB (typically found to be 13-17dB).

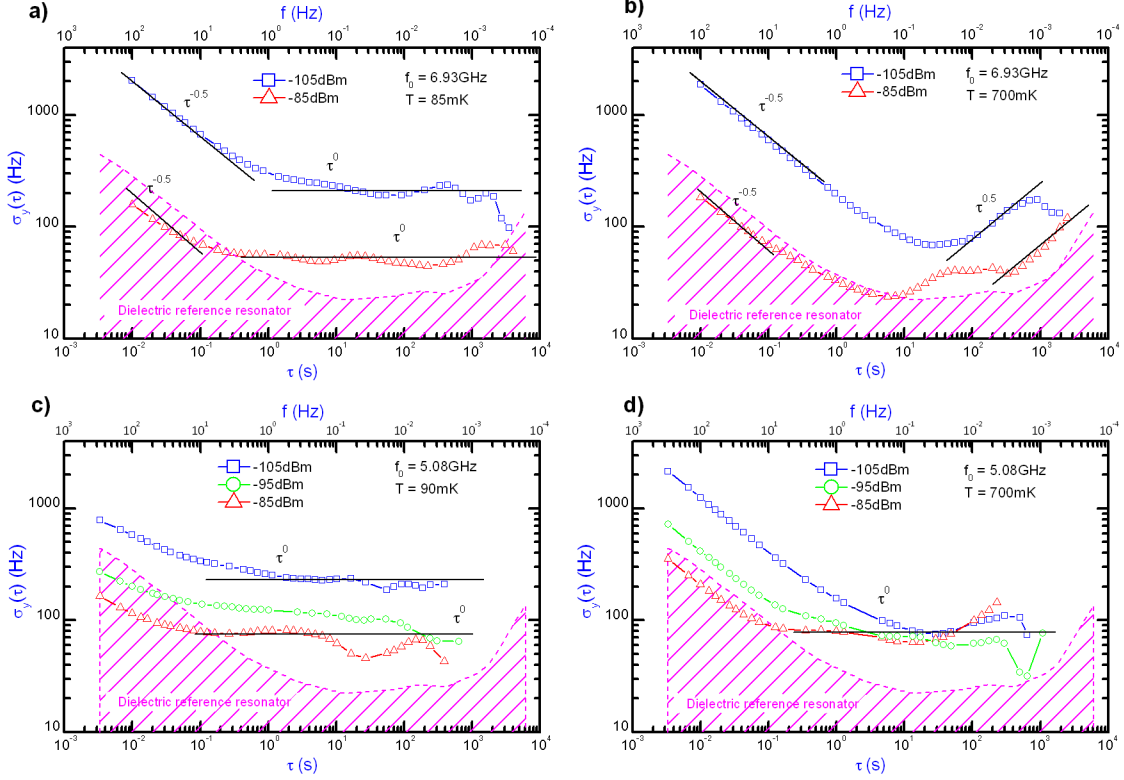


Figure 2. Frequency stability as a function of measurement integration time. Successive shapes correspond to different microwave drive powers, triangles = -85 dBm, circles = -95 dBm and squares = -105 dBm. Plots a) and b) are of a bare resonator, B1, and an aluminium oxide covered resonator, A1, in plots c) and d) where the temperature is 90 mK for plots a) and c) and 700 mK for plots b) and d).

Applied microwave drive varies between a high power of -85 dBm and a low power of -105 dBm corresponding to circulating powers  $P_{circ} \approx -40$  dBm to  $P_{circ} \approx -60$  dBm. Under high applied drive the circulating power exceeds the saturation power [12] and is a typical power for kinetic inductance detector applications [20]. At such a high power, many resonant TLFs should be saturated, leaving the frequency shift to be caused by non-resonant TLFs [18]. While the low microwave drive corresponds to an average of approximately 100 photons within the resonator and should be near the limit for completely de-saturated resonant TLFs.

At low temperatures and low microwave drive the behaviour of both the bare and covered resonators is similar (square traces in figures 2a and 2c). There are two dominant regions in these plots, at short times a large jitter is present due to a frequency independent noise process which manifests as white frequency noise (noise obeys a power law of  $\tau^{-0.5}$ ) which predominately is caused by the instrumentation [21]. The second region occurs at long time scales from 1 second here we directly observe flicker frequency noise (noise obeying a power law of  $\tau^0$ ) which

limits frequency stability in bare resonators to  $\approx 200$  Hz and in covered resonators to  $\approx 300$  Hz.

When increasing the microwave drive the entire noise level is observed to decrease for both the bare and covered resonators (triangle traces in figures 2a and 2c). The noise processes remain the same, white frequency noise at short times and flicker frequency noise at long time scales. Increased microwave drive saturates the resonant TLFs leading to a reduction in the flicker frequency noise level by a factor of  $\approx 3$ . At 90 mK the temperature is not sufficient to excite TLFs, in figures 2b and 2d measurements are performed at 700 mK where TLFs can be thermally excited.

At higher temperatures and low microwave drive the behaviour of covered resonators is similar to the low temperature with white frequency noise dominating at short times and flicker frequency noise dominating at times in excess of 10 seconds. Compared to the bare resonator, white frequency noise still dominates at short times but the longer time scales now exhibit random frequency walk noise (noise obeys a power law of  $\tau^{0.5}$ ). The lack of flicker frequency noise leads to a reduced noise level with mini-

imum of 90 Hz when averaging for 30 seconds. At higher microwave drive the flicker frequency noise level of the covered resonator remains the same, while the bare resonator sees a consistently lower noise level, again without flicker frequency noise. The lowest noise level is found to be 30 Hz, corresponding to a fractional frequency shift of 4 parts in  $10^9$  when averaging for 5 seconds.

These resonators are shown to directly exhibit flicker frequency noise, consistent with the resonator coupling to a bath of two level fluctuators, if the TLF ensemble is coherent and weakly interacting with the resonator such a bath has been theoretically shown to produce a flicker frequency spectral noise[4]. The noise level is also found to increase with increasing loss tangent, consistent with noise increasing with increasing density of TLFs[4]. At low temperatures there exists a power dependence of the noise which could scale similar to the  $P_{int}^{-1/2}$  dependence suggested by Gao et al[22] however more measurements are required to verify the exact power dependence.

At higher temperatures the flicker frequency is not observed for bare resonators, considering the loss tangent data, we expect 700 mK to exceed the  $\Delta_{max}$  energy scale of the TLFs determined by Shnirman et al[4], resulting in the resonator no longer coupling to the coherent TLF ensemble and hence the resonator not exhibiting flicker frequency noise. A possible candidate for the new ran-

dom walk noise process is instead thermally activated flux motion. Within the covered resonator flicker frequency noise exists even at 700 mK, however the level is no longer power dependent. It is possible that the surface TLFs have a larger  $\Delta_{max}$  than those in the Niobium-Sapphire interface, but this should appear within the loss tangent measurements. Instead the authors suspect the resonator couples to a coherent bath of TLFs, which are themselves affected by the incoherent action of other TLFs as was recently suggested by Faoro and Ioffe[3] and observed by Grabovskij[2]. The action of "slow" fluctuators limits the RMS deviation of the resonance frequency over large time scales  $\sim 1$  second which leads to the observed flicker frequency noise. Since these "slow" fluctuators are not coherently coupled to the resonator, they are not affected by microwave drive.

To conclude we directly observed slow processes in superconducting resonators and studied their dependence on power, temperature and TLF density. Such processes are described in theory[3], shown to affect qubits[2] and are expected to affect other superconducting or charge sensitive devices such as kinetic inductance detectors.

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- [1] Koch, R. H., Van Harlingen, D. J., and Clarke, J. *Phys. Rev. Lett.* **45**, 2132–2135 Dec (1980).
  - [2] Grabovskij, G. J., Peichl, T., Lisenfeld, J., Weiss, G., and Ustinov, A. V. *Science* **338**(6104), 232–234 (2012).
  - [3] Faoro, L. and Ioffe, L. B. *Phys. Rev. Lett.* **109**, 157005 Oct (2012).
  - [4] Shnirman, A., Schön, G., Martin, I., and Makhlin, Y. *Physical Review Letters* **94**(12), 1–4 April (2005).
  - [5] Lindström, T., Burnett, J., Oxborrow, M., and Tzalenchuk, A. Y. *Review of Scientific Instruments* **82**(10), 104706.
  - [6] Healey, J. E., Lindström, T., Colclough, M. S., Muirhead, C. M., and Tzalenchuk, A. Y. *Applied Physics Letters* **93**(4), 043513 (2008).
  - [7] Bothner, D., Gaber, T., Kemmler, M., Koelle, D., and Kleiner, R. *Applied Physics Letters* **98**(10), 102504 (2011).
  - [8] Wang, H., Hofheinz, M., Wenner, J., Ansmann, M., Bialczak, R. C., Lenander, M., Lucero, E., Neeley, M., OConnell, a. D., Sank, D., Weides, M., Cleland, A., and Martinis, J. M. *Applied Physics Letters* **95**(23), 233508 (2009).
  - [9] Gao, J. R., Daal, M., Vayonakis, A., Kumar, S., Zmuidzinas, J., Sadoulet, B., Mazin, B. a., Day, P. K., and Leduc, H. G. *Applied Physics Letters* **92**(15), 152505 (2008).
  - [10] Vissers, M. R., Kline, J. S., Gao, J. R., Wisbey, D. S., and Pappas, D. P. *Applied Physics Letters* **100** (2012).
  - [11] Lindström, T., Healey, J. E., Colclough, M. S., Muirhead, C. M., and Tzalenchuk, A. Y. *Phys. Rev. B* **80**, 132501 Oct (2009).
  - [12] Macha, P., van Der Ploeg, S. H. W., Oelsner, G., Ilichev, E., Meyer, H.-G., Wunsch, S., and Siegel, M. *Applied Physics Letters* **96**(6), 062503 (2010).
  - [13] Wenner, J., Barends, R., Bialczak, R. C., Chen, Y., Kelly, J., Lucero, E., Mariantoni, M., Megrant, A., Malley, P. J. J. O., Sank, D., Vainsencher, A., and Wang, H. *Applied Physics Letters* **99**(2011), 1–4 (2011).
  - [14] Rubiola, E. *Phase Noise and Frequency Stability in Oscillators*. Cambridge University Press, Cambridge, England, (2009).
  - [15] Mattis, D. C. Bardeen, J. *Physical Review Letters* **1**(11), 407–408 December (1958).
  - [16] Von Schickfus, M. and Hunklinger, S. *Physics Letters* **64A** (1977).
  - [17] Strom, U., von Schickfus, M., and Hunklinger, S. *Phys. Rev. Lett.* **41**, 910–913 Sep (1978).
  - [18] Pappas, D. P., Member, S., Vissers, M. R., Wisbey, D. S., Kline, J. S., and Gao, J. R. *IEEE Transactions on Applied Superconductivity* **21**(3), 871–874 (2011).
  - [19] Hartnett, J. G., Nand, N. R., and Lu, C. *Applied Physics Letters* **100**(18), 183501 (2012).
  - [20] Leduc, H. G., Bumble, B., Day, P. K., Eom, B. H., Gao, J. R., Golwala, S., Mazin, B. a., McHugh, S., Merrill, A., Moore, D. C., Noroozian, O., Turner, A. D., and Zmuidzinas, J. *Applied Physics Letters* **97**(10), 102509 (2010).
  - [21] *In some cases the white noise level of the dielectric resonator exceeds those of superconducting resonator. This is due to the difference in insertion loss.*
  - [22] Gao, J. R., Daal, M., Martinis, J. M., Vayonakis, A., Zmuidzinas, J., Sadoulet, B., Mazin, B. a., Day, P. K., and Leduc, H. G. *Applied Physics Letters* **92**(21), 212504 (2008).